Math 1C Quiz 4

Tue Jun 15, 2021 DUE Thu Jun 24, 2021 @ noon in Canvas

SCORE: _____ / 35 POINTS

INSTRUCTIONS

[A] For this test, you may consult your lecture notes for sections 12.1-12.5, 13.1-13.4, the Zoom recordings for sections 12.1-12.5, 13.1-13.4 linked on the instructor's website, your textbook sections 12.1-12.5, 13.1-13.4 and the homework you did for those sections. Unless otherwise specified, you may **not** simply use the answer of a textbook exercise as justification – you must write a complete solution for that answer in your work.

You may **not** use any other material located on the instructor's website nor covered in any other sections of your textbook.

You may **not** consult any person, nor any written/printed material, website, software, app or other electronic resource, nor any calculator (unless instructed), computer, phone or other electronic device.

[B] For each derivative that requires the product, quotient and/or chain rule, you must show all the factors and terms from the rule before you simplify.

eg. $\dfrac{d}{dx}\dfrac{xe^{2x}}{1+e^x} = \dfrac{(1(e^{2x})+xe^{2x}(2))(1+e^x)-xe^{2x}(e^x)}{(1+e^x)^2}$ ⬅ **THIS STEP IS REQUIRED**

$= \dfrac{e^{2x}((1+2x)(1+e^x)-xe^x)}{(1+e^x)^2} = \dfrac{e^{2x}(1+2x+e^x+xe^x)}{(1+e^x)^2}$

For each integral that requires $u-$substitution, you must show the value of u and the resulting integral in terms of u.

eg. $\int 3x(2x^2-1)^5\,dx$ $= \int \frac{3}{4}u^5\,du \quad (u=2x^2-1)$ $= \frac{1}{8}(2x^2-1)^6+C$ ⬅ **MINIMUM WORK REQUIRED**

For each integral that requires integration by parts, you must show the work (table method OK).

For each integral that requires polynomial long division, partial fractions decomposition or trigonometric substitution, you must show all work.

The general rule is that, **if you can't do the work in your head without writing something down, write it in your test**.

[C] **Handwrite** your solutions to the questions on clean 8½″ × 11″ paper (or equivalent).

[D] Your solutions to the questions must be in the same order as the questions in this test. (You may write the solutions to each question on separate pages, and sort them in order afterwards.)

[E] You do not need to copy the questions onto your paper. Just show your organized and clearly written work and final answers.

[F] Writing which is illegible to the instructor will earn 0 points.

[G] All final algebraic answers must be completely simplified to receive full credit.

[H] All work must be properly algebraically justified and use proper mathematical notation as shown in lecture, not simply based on scratch work, intuition or "handwaving".

Imagine yourself tutoring a struggling Math 1C student. They should be able to understand your algebraic reasoning based on your writing alone.

[I] Upload a **single** clear & legible PDF of your completed test to Canvas no later than Thu Jun 24 @ noon Pacific Time.

The solution will be posted to my website shortly after that time, and all work submitted after that will earn 0 points.

QUESTIONS

[1] Legibly write the text from the box below (do **not** write in cursive), and **sign your name directly below your writing**.
If you skip this step or your writing is illegible to me, your test will not be counted for credit.

> "I am a principled and honorable person who can be trusted.
> I pledge to uphold the De Anza College Student Code of Conduct.
>
> My signature confirms that the work shown on this test is strictly my own.
> Other than the resources listed in Instruction [A] of this test,
> I did not consult any person, nor any printed/written material, website, software, app or other electronic resource,
> nor any calculator, computer, phone or other electronic device."

NOTE: The De Anza College Student Code of Conduct can be found at
https://go.boarddocs.com/ca/fhda/Board.nsf/goto?open&id=9U2UC77B2DA5

[2] **[2 POINTS]** Let $\vec{r}(t)$ be a vector function such that $\vec{B}(t)$ is never $\vec{0}$. Find $\frac{d}{dt}\| \vec{B}(t) \|$.

NOTE: To receive credit, you must provide a brief explanation for your answer.
HINT: This question is worth very few points.

[3] **[2 POINTS]** Let $\vec{r}(t)$ be a vector function corresponding to the intersection of $3x-4y+1=0$ and $x^2+y^2+z^2=12$.
Find $\vec{r}(t)\cdot\vec{r}\,'(t)$.

NOTE: To receive credit, you must provide a brief explanation for your answer.
HINT: This question is worth very few points.

[4] **[4 POINTS]** Let $\vec{r}(t) = < t, 1, 0 >$.

[a] Find $\left\| \frac{d}{dt}\vec{r}(t) \right\|$.

[b] Find $\frac{d}{dt}\| \vec{r}(t) \|$.

[c] Does $\frac{d}{dt}\| \vec{r}(t) \| = \left\| \frac{d}{dt}\vec{r}(t) \right\|$? (ie. does $\| \vec{r}\,'(t) \| = \| \vec{r}(t) \|'$?)

[5] **[7 POINTS]** Simplify the expression $\frac{\vec{r}\,' \times (\vec{r}\,'' \times \vec{r}\,')}{\| \vec{r}\,' \|^3}$ using vector algebra by following the steps below.

You are strongly encouraged to use parts of the proof of Section 13.3 Theorem 10 on pages 864-865, as well as various vector algebra properties from chapter 12, to reduce your work.
NOTE: You must state, but do not need to reprove, the parts of the textbook's proof that you use.
Do **not** use component form in your work.

[a] Find an expression for $\vec{r}\,'' \times \vec{r}\,'$ using $\vec{r}, \vec{r}\,', \vec{T}$ and/or $\vec{T}\,'$.
Your final answer must **not** use s nor negatives. Justify briefly each step of your work.

[b] Using the answer of [a], find an expression for $\frac{\vec{r}\,' \times (\vec{r}\,'' \times \vec{r}\,')}{\| \vec{r}\,' \|^3}$ using cross products involving $\vec{T}$ and $\vec{T}\,'$.
Your final answer must **not** use $s, \vec{r}$ nor $\vec{r}\,'$. Justify briefly each step of your work.

[c] Completely simplify the answer of [b]. Justify briefly each step of your work.

[6] **[20 POINTS]** Consider the point $P(1, 0, 2)$ on the space curve $\vec{r} = < e^t, -2t, 2e^{-t} >$.

[a] Find parametric equations of the tangent line to the curve at P.

[b] Find the linear (simplified) equation of the normal plane to the curve at P.

[b] Find $\vec{T}$, $\vec{N}$ and $\vec{B}$ at P.

[c] Find the curvature at P.

[d] Find the equation of the osculating plane at P.

[e] Find the center and radius of the osculating circle at P.